20th IAHR International Symposium on Ice
Lahti, Finland, June 14 to 18, 2010

Progress in Shallow Water Ice Profiling Sonar (SWIPS) for River and Lake Ice Monitoring

Jan Buermans, David B. Fissel, **John R. Marko**
ASL Environmental Sciences Inc.
Sidney, B.C., Canada
jbuermans@aslenv.com
and
Martin Jasek,
BC Hydro
Burnaby, B.C., Canada
Martin.Jasek@bchydro.com

Since first introduced, in 2004, into annual river ice monitoring programs on the Peace River, Shallow Water Ice Profiling Sonar (SWIPS) has been developed for quantitative monitoring and study of ice in river, lake and laboratory environments. This development will be reviewed, beginning with identifications of the environmental features of interest and proceeding to: descriptions of the instrument; deployment and operating principles; and output products. A key step in this advancement was the recognition of capabilities for detection of suspended frazil ice which, in combination with known capacities for draft and other measurements on floating surface ice, raised prospects for reasonably complete quantitative tracking of all components of river and lake ice systems.

An overview is provided on the effective extraction of information for different components of the ice environment. The selection and use of different acoustic frequencies in the SWIP is discussed and shown to be an important tool for extracting optimal information on river ice processes. For frazil ice pans and for the seasonal ice cover and its usual layer of slush ice, precision in establishing the location of ice/water boundaries increases with the acoustic frequency. On the other hand, the actual acoustic penetration of the ice cover, and, hence, possibilities for characterizing the underside of the ice cover, especially slush ice, become more effective with the use of lower acoustic frequencies. This topic is being addressed through ongoing analysis and research.

1. Introduction and background

Ice Profiling Sonar (IPS) has been widely used in the Polar ocean regions since 1990 (Melling et al., 1995). ASL has pioneered in the evolution of the acoustic profiling instruments used in this work, recently introducing a fifth generation Ice Profiler instrument (IPS5), which sets the current global standard for autonomous sea ice thickness measurement in remote areas. Applications of the basic profiling technology in fresh- as opposed to salt-water areas were first made on the St. Lawrence River during the 2002-2003 winter season (Chave, et al. 2004).

First introduced in 2004 into annual river ice monitoring programs on the Peace River (Jasek, 2005), the Shallow Water version of the Ice Profiling Sonar (SWIPS), intended for ice measurements in fresh water, has been developed for qualitative and quantitative monitoring and study of ice in river, lake and laboratory environments.

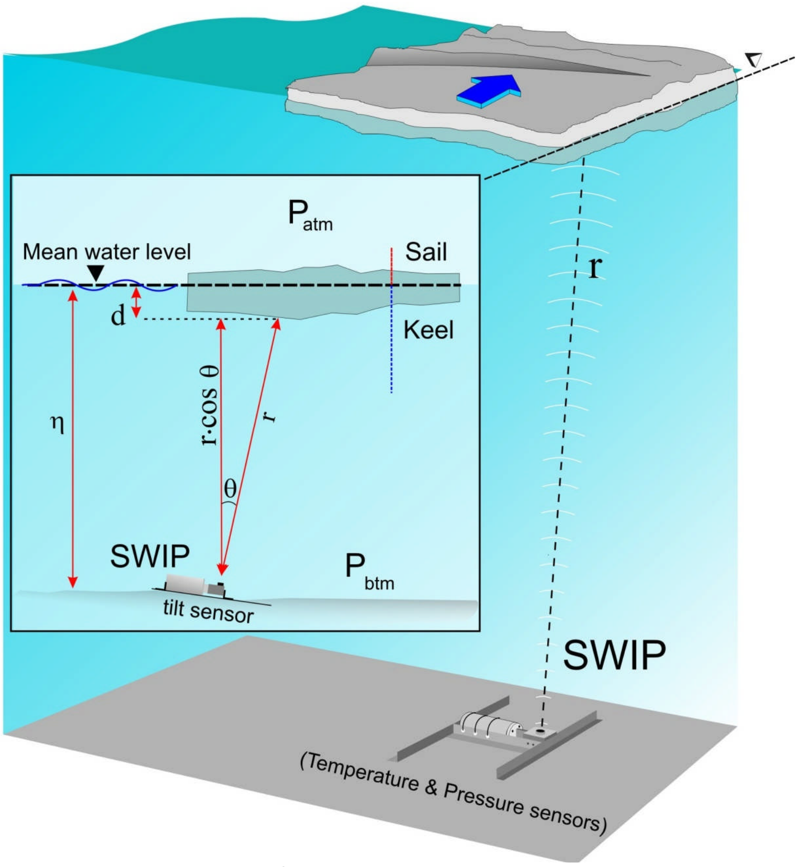


***Figure 1**:* SWIP configuration and measurements parameters

Frazil ice is a collection of loose, randomly oriented ice crystals in water. Frazil ice is the first stage in the formation of ice and it forms in open, turbulent and supercooled water at low air temperatures. Hydrodynamic drag in turbulent and fast flowing water can distribute the small crystals throughout the water column. As the ice crystals increase in size and also start to stick together, buoyancy forces will become more important and will carry the crystals to the surface. When these larger ice crystals coagulate and concentrate at the air-water interface (or at the water-ice cover interface), this mixture of ice and water is referred to as slush ice.

In the Peace River, large ice crystals in slush ice at the surface coalesce to form ice pans. During the ice cover formation process or freeze-up, these ice pans are brought to a halt at a constriction point and form an initial ice cover consisting of juxtaposed pans and slush ice 0.5 m to 1.0 m in thickness. The ice cover can consolidate further due to ice shoving events that can thicken the slush ice cover 2 to 5 m in thickness. With the progression of winter, the underside of the ice cover erodes as slush ice is carried away by the current. The thermal ice is considered the solid portion of the ice cover that forms as the slush voids freeze. Typically over the course of the winter, the thermal ice reached a thickness around 0.8 m.

BC Hydro initially deployed a 235 kHz SWIPS instrument for the 2004-05 winter and the next year, BC Hydro added the high-frequency 546 kHz SWIP to the program. During the 2007-08 winter, an improved design of the heated anchor-ice resistant mooring worked successfully. Data from both a high and low frequency SWIP instrument were available in real-time via a secure website for BC Hydro operations for a portion of the 2008-09 winter.

This paper outlines the basics of the SWIPS technology in terms of progressive refinements in instrument and deployment technologies applied in BC Hydro's Peace River programs over the 2004-2009 period. Key items in these programs were the progressive exploitation of the dependences of detected target returns on acoustic frequency and the development of heated anchor-ice resistant instrument mountings equal to the task of surviving episodes of explosive anchor ice growth in pre-freeze-up supercooling periods.

2. Principles of Operation

The SWIP has been designed as a self-contained or real-time instrument to observe stationary or moving ice through the sonar's field of view (Figure 1). Since SWIP shares its software, firmware, and main electronic components with the IPS, its principles of operation are identical. The SWIP sensors and housing configuration have been optimised for deployments depths of 2-20 m below the surface, looking upward. At intervals set by the deployment software, the acoustic transducer transmits a pulse of programmable duration. The sound travels in the form of a conical beam toward the surface. Some of the sound is absorbed as it travels through the water and some of the sound is reflected by frazil, slush, and thermal ice, the water-air interface or other targets it may encounter. A short time after the pulse is transmitted, the acoustic transducer starts listening for these reflections (echos) from the water column. The voltage signal generated by the transducer is amplified to account for spreading and absorption losses and it is then digitized by the analog-to-digital (A/D) converter. This digitized voltage output is referred to as the return strength. With the standard digitization rate of 64 kHz, the instrument has a time resolution of about 16 microseconds (or 25 microseconds if the digitization rate is set to 40 kHz). The attainable resolution in space is of the order of 1 cm.

In target mode, which is designed specifically for ice draft measurements, the SWIP examines the return strength from echoes. Using a well-established algorithm, the instrument then decides which part of the signal is returned from the bottom of the ice, or in the absence of ice, from the water-air interface. The interval between transmission and receipt of the selected target is referred to as the Travel Time. This parameter is measured and recorded internally onto removable CompactFlash memory or transmitted in real-time over the serial interface. The SWIP can record the maximum Amplitude, the duration (Persistence) and Travel Time of up to a maximum of 5 detected targets. The Travel Time is then used to compute the range (r) to the detected target using a best estimate of the water speed of sound.

In profiling-mode, the SWIP records the return strength, expressed in Counts, for the entire water column. This series of values represents the acoustic backscatter from the water column and can be used to detect frazil ice, investigate slush and explore the ice cover, as well as detect the surface ice and the air/water interface (if present). With the application of the speed of sound, the return strength versus range can be plotted over a period of time.

The SWIP also contains a real time clock and sensors for measuring instrument tilt from vertical on two axes, for measuring pressure and for measuring temperature. The frequency of the recording of these auxiliary variables may be controlled by the user. The tilt data allow the calculation of zenith distance from return range (Figure 1). The pressure sensor allows for calculation of the actual depth of the SWIP beneath the water level, as the instrument's depth below water level changes in response to current, set up from wind and possible mooring motion. To complete this calculation, atmospheric pressure must be determined by means of independent instrumentation at the surface, as discussed in more detail below.

The SWIP may be programmed to start data collection immediately, or to wake up at a future time. It may also be programmed to go to sleep for a specific time. It can store acquired data in its removable commercially-available CompactFlash memory and/or send the data over a serial communication port. For real-time communication over a long underwater cable, the instrument is usually supplied with an RS-422 serial communications port.

3. The SWIP Mooring Arrangement

On the Peace River, successful operations of SWIPS instrumentation in the presence of anchor ice have posed considerable challenges over the years. Prior to ice cover consolidation, frazil ice (as anchor ice) can cover up the acoustic transducer blocking the acoustic energy and/or attach itself to mooring frame. If it builds up in sufficient quantity, the buoyancy of the deposited anchor ice can turn over the mooring or lift it completely. Once lifted into the current, the system can readily be damaged and or lost permanently. This is exactly what happened during 2006-07 winter season. The redesigned heated pyramid mooring with a single mooring line (shown in Figure 2) was successful in subsequent seasons.

***Figure 2**:* SWIPS Deployment on Peace River during the fall of 2007

***Figure 3**:* SWIPS with internal battery being deployed

For applications where anchor ice does not pose any challenges, users have mounted the SWIP in a simple mount as shown in figure 3 (*picture courtesy of Dr. Yasuhiro Yoshikawa, Civil Engineering Research Institute for Cold Region, Sapporo, Hokkaido, Japan*). In deeper water, it is possible to use the taut-line mooring configuration shown in figure 4 to place the instrument within 20m from the surface. For installations without a cable, it is useful to use one or more of the following to aid in the recovery: acoustic pinger locator, acoustic release, weighted-down polypropylene ground line and/or pop-up buoy. A combination of these recovery aids is recommended since instruments have occasionally been moved or lost entirely in a dynamic environment.

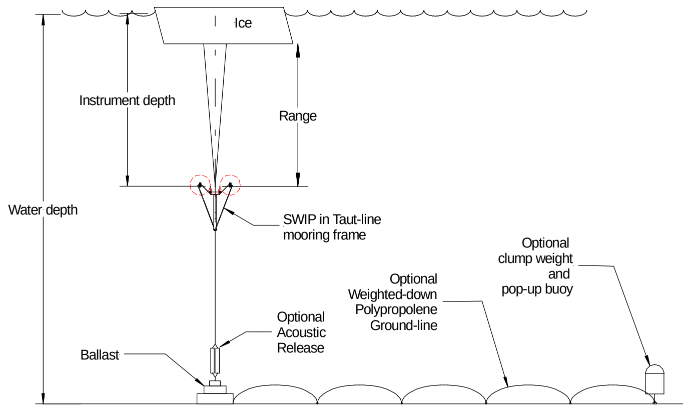


***Figure 4:** Possible mooring arrangement for the Shallow Water Ice Profiler in deeper water*

4. Ice draft and ice thickness computations from SWIP data –Target mode

Ice draft computation, using data obtained from an upward looking sonar deployed in a saltwater environment, is well understood (Fissel et al. 2008). The SWIP parameters measured include Travel Time (from which range(r) to the underside of the ice is computed), tilt in two planes ($tilt_x$ and $tilt_y$), absolute water pressure at the instrument (P_{btm}), and near-bottom water temperature. Barometric pressure (P_{atm}) needs to be measured separately. It may also be useful to conduct water column measurements of the speed of sound while the body of water is ice covered. To compute ice draft, this information needs to be entered into Eq. 1 thru 4 below. For salt water applications (Marko, 2006), the critical, accuracy-limiting factor in ice profiling is knowledge of the mean sound speed. The actual speed of sound is only available with accuracy over the full water column at the start and end of a deployment through direct conductivity-temperature-density (CTD) profile measurements for temperature and salinity. For intermediate times, sound speed estimates are obtained as an integral part of the data processing/analysis program. This is done by establishing values of β (Eq.4) which correctly yield zero draft values from Eq. 3 using range, r, and water level, η, values at times when there is unambiguous presence of open water above the SWIP instrument.

Table 3: Speed of sound in fresh water as a function of water temperature

Water Temperature (°C)	Sound Speed (c in m/s)
0	1402.39
1	1407.37
2	1412.23
3	1416.99
4	1421.63
5	1426.16
6	1430.59
7	1434.91
8	1439.13
9	1443.25

$$\eta = (P_{btm} - P_{atm})/ \rho g - \Delta D \quad (1)$$

Where: η is the water depth above the acoustic transducer

P_{btm} is the hydrostatic (bottom) pressure as measured by the SWIP

P_{atm} is the atmospheric pressure

ρ is density of water

g is acceleration of gravity

ΔD is physical separation in the vertical direction between the deployed acoustic and hydrostatic pressure sensors

$$\theta = (tilt_x^2 + tilt_y^2)^{1/2} \quad (2)$$

where: θ is the tilt magnitude with respect to the vertical

$tilt_x$ is the measured tilt angle in the x-plane

$tilt_y$ is the measured tilt angle in the y-plane

$$d = \eta - \beta \cdot r \cdot \cos\theta \quad (3)$$

where: d is the ice draft

β is a "*to be determined*" factor which accounts for changes over time in the mean sound speed in the upper water column

r is the range to the ice as measured by the acoustic transducer

$$\beta = c_{act} / c_{IPSLink} \quad (4)$$

where: c_{act} is the actual mean speed of sound in the water column and

$c_{IPSLink}$ is the speed of sound entered in the IPS*Link* software for the instrument deployment.

Without a snow cover and with relatively flat ice, the ice thickness (T_{ice}) can be estimated if the densities of water (ρ_{water}) and ice (ρ_{ice}) are known:

$$T_{ice} = d \cdot \rho_{water}/\rho_{ice} \quad (5)$$

For fresh water applications, corrections for the time varying speed of sound are more straight forward since the mean sound speed is entirely a function of water temperature. During the cooling and ice covered seasons, the measured water temperatures can be used to directly determine the speed of sound of freshwater to a reasonable degree of accuracy. Table 3 lists the speed of sound in fresh water for water temperatures from 0 to 9°C. After ice break-up, thermal stratification in the water column above the SWIP instrument may require the use of the open water episodes to determine the speed of sound, as described above for sea water computations.

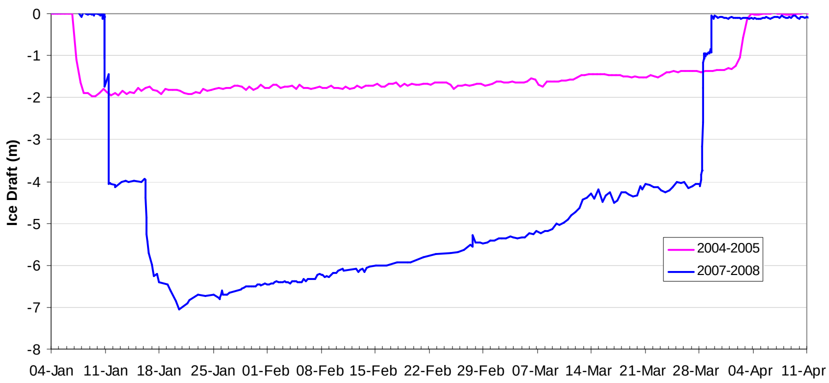


***Figure 5**:* Ice draft time series for the 2004-2005 and 2007-2008 study program from the Peace River SWIP deployments.

Figure 5 presents time series plots of ice drafts for two winter seasons on the Peace River and includes the slush layer.

5. Acoustic observation and measurement of the slush layer in rivers –Profile mode

In the case of the mixed ice cover (thermal ice, slush ice and frazil ice) of the Peace River, the slush layer under the thermal ice generally is limited to 1-2 metres thickness over the winter but can erode to 0.5 to 1.5 metres in the latter months of the ice covered season. Locally, there can be areas where slush is being eroded more rapidly or is even being deposited depending on the water velocity. The ice drafts measured by the SWIP include the frazil slush layer if it is present.

Slush is mostly a mixture of water, ice crystals, and air bubbles and its acoustic properties, including its intrinsic speed of sound, can be expected to exhibit considerable variability. Some attempts have been made to measure the speed of sound within the slush ice in the field and results appear variable. We suspect the speed of sound in slush is 10 to 20% less than in clear fresh water. Once the sound travels into the slush, the speed of sound and attenuation can be variable and interpretation of backscatter from within the slush ice becomes more difficult. The physical properties of slush ice are the subject of further collaborative research studies.

The reflected acoustic power can be quantified and derived from the amplified transducer voltage measured. In absolute terms, the links between the voltage output and the acoustic power detected from any given range are quantifiable in terms of known or measureable system parameters such as gains, A/D conversion rates, driving voltages and transducer transmitting and receiving efficiencies. A comparison of detected acoustic power relative to the similarly-calculated "outgoing" acoustic power at the range of the target provides the basis for ice or other target characterization. The Addendum entitled: "Converting counts to absolute backscatter and sound intensity -The Sonar Equation" covers this topic in some detail. However, as a simple tool to classify ice covers, it is useful to introduce the concept of relative sound intensity as equal to the square of the voltage (i.e. in Counts^2):

$$\textit{relative intensity} = \text{Counts}^2/\text{Counts}_{max}^2 \quad \text{or} \quad \text{Counts}^2/65{,}535^2 \;\; \textit{for 16-bit A/D} \tag{6}$$

When targets correspond to distinct surfaces such as the underside of a moving or stationary ice cover above a relatively "target free" water column, the SWIP intensities rise sharply (Figure 6). These intensities extend either to a high level and then drop off sharply (a hard surface) or first level off at an intermediate values before rising again to the maximum intensity level. The second of these cases corresponds to soft surface returns which are most characteristic of a stationary ice cover containing a slushy lower layer. This slush layer gives weaker returns prior to later arrivals of stronger returns from a hard or thermal ice interface positioned higher in the ice cover interior. In both cases, the actual intensities and the fractions of the incident acoustic power that they represent are relatively complicated functions of the ice properties such as roughness and porosity as well as sound frequency.

Most SWIP ice cover applications do not attempt to draw quantitative measures of ice properties but focus on draft estimation. However, lower acoustic frequencies (longer wave lengths) penetrate the lower ice cover and appear to allow useful extraction of qualitative or semi-quantitative data on changes inside the slush layer. These changes are likely to be significant in characterizing the seasonal ice cover and its interactions with water column frazil ice. This topic is the subject of further study in collaboration with research partners and SWIP users.

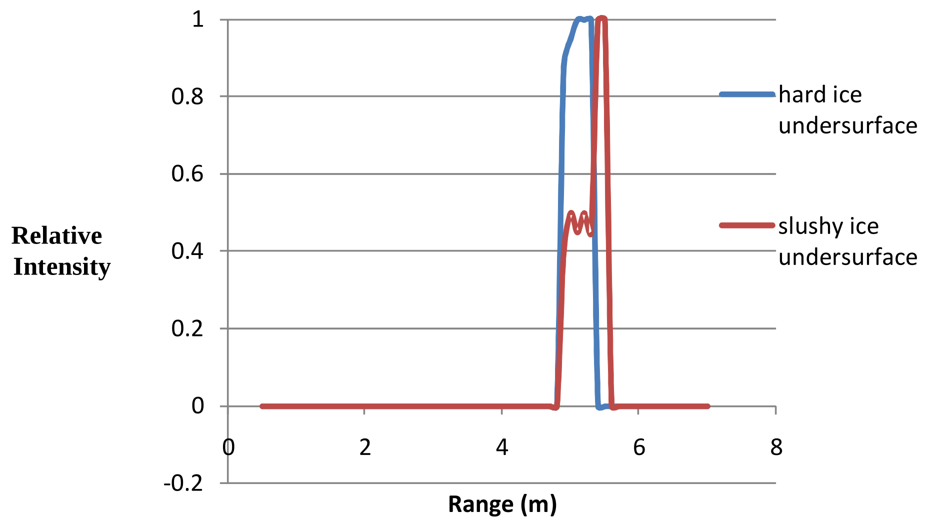


Figure 6: Schematic representation of relative SWIPS intensity as a function of range above a SWIPS transducer under ice covers with hard and soft (slushy) lower surfaces.

6. Frazil Observations and Monitoring –Profile Mode

In profile mode, the SWIP can be used to record the volume backscatter from distributed scatterers in the water column such as frazil ice and ice fragments. Pre-freezeup return strength from the 546 kHz and from the 235 kHz SWIP instruments are shown in figures 7 and 8 as a function of range over a period of time.

The color shown indicates the raw counts recorded on a scale of 1-255 (8-bit A/D) in this case. In comparing the two graphs representing the same time period, it is apparent that the 546 kHz frequency is better at detecting the fine frazil crystals than the 235 kHz frequency. The higher acoustic frequency, because of its associated smaller wavelength, is much more effective in producing larger return strengths from very small frazil crystal targets. The use of multiple frequency SWIP acoustic transducers, along with more advanced analysis methodology, provides a powerful research tool for frazil ice studies (Marko and Jasek, 2010a; 2010b).

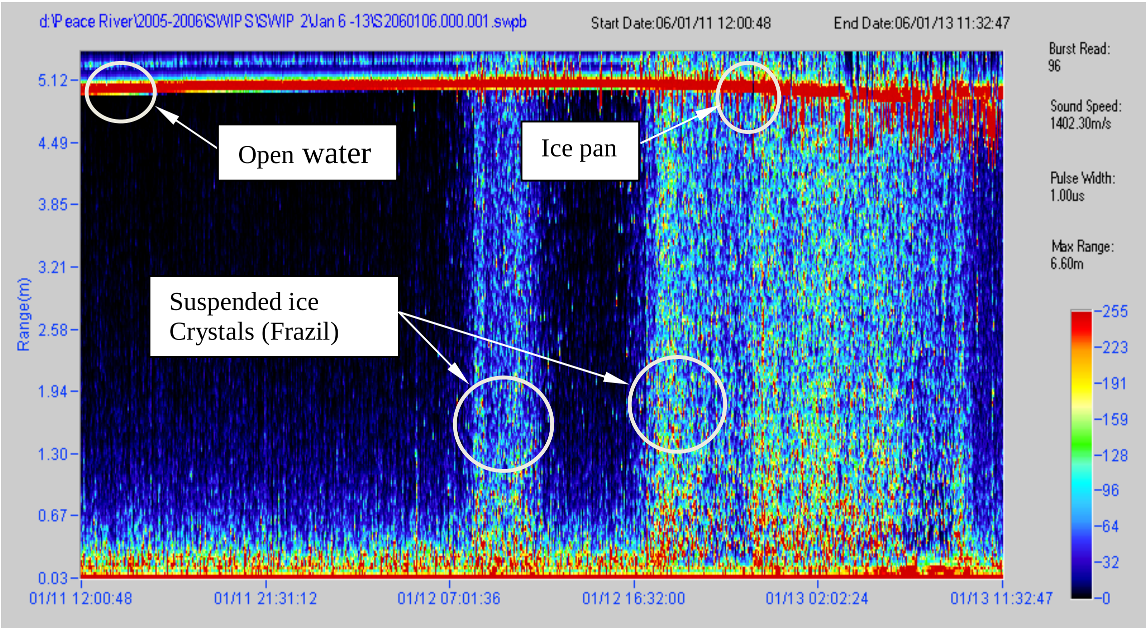


Figure 7: **546 kHz** unit 2006 Peace River 24 hours of profile data- High frequency results show return stength variability in same intervals associated with frazil ice presence prior to freeze up (depths less than 4.8 m). (Same period as Figure 8) (Ref: Marko et al, 2008)

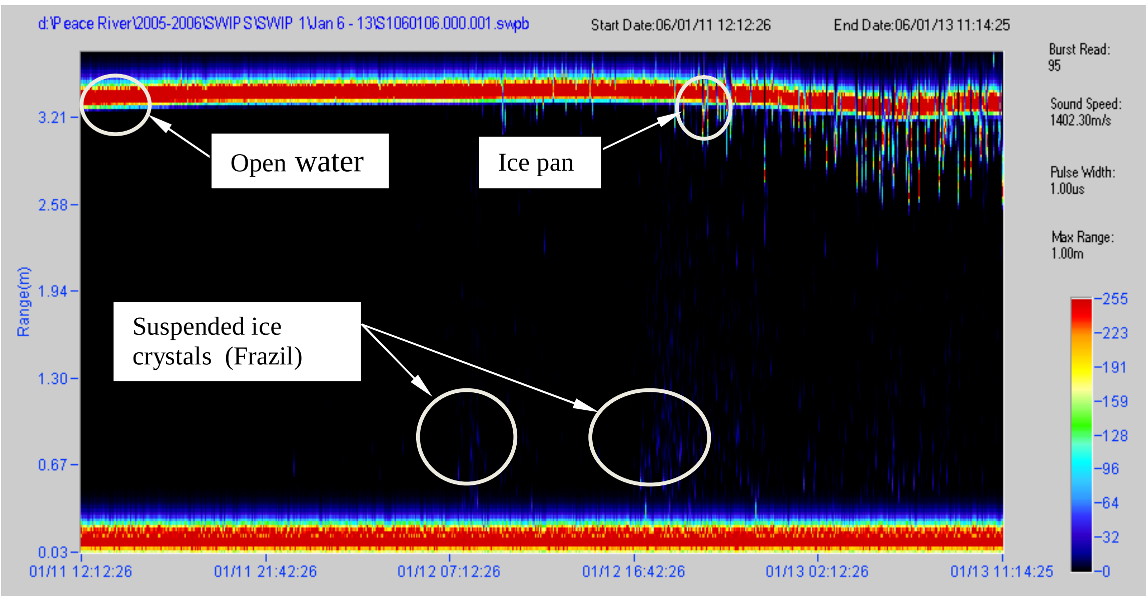


Figure 8: **235 KHz** unit 2006 Peace river 24 hours of profile data results show weak return strength (depths less than 2.94 m) appearing primarily after 16:30 1/12 prior to freeze up. (Same period as Figure 7) (Marko et al. IAHR 2008)

Episodes of moving slush ice have been observed as dense mass moving under the midwinter ice cover with the 235 kHz SWIP in the Peace River (See Figure 9). The colours in the diagram show Counts ranging from 1-255.

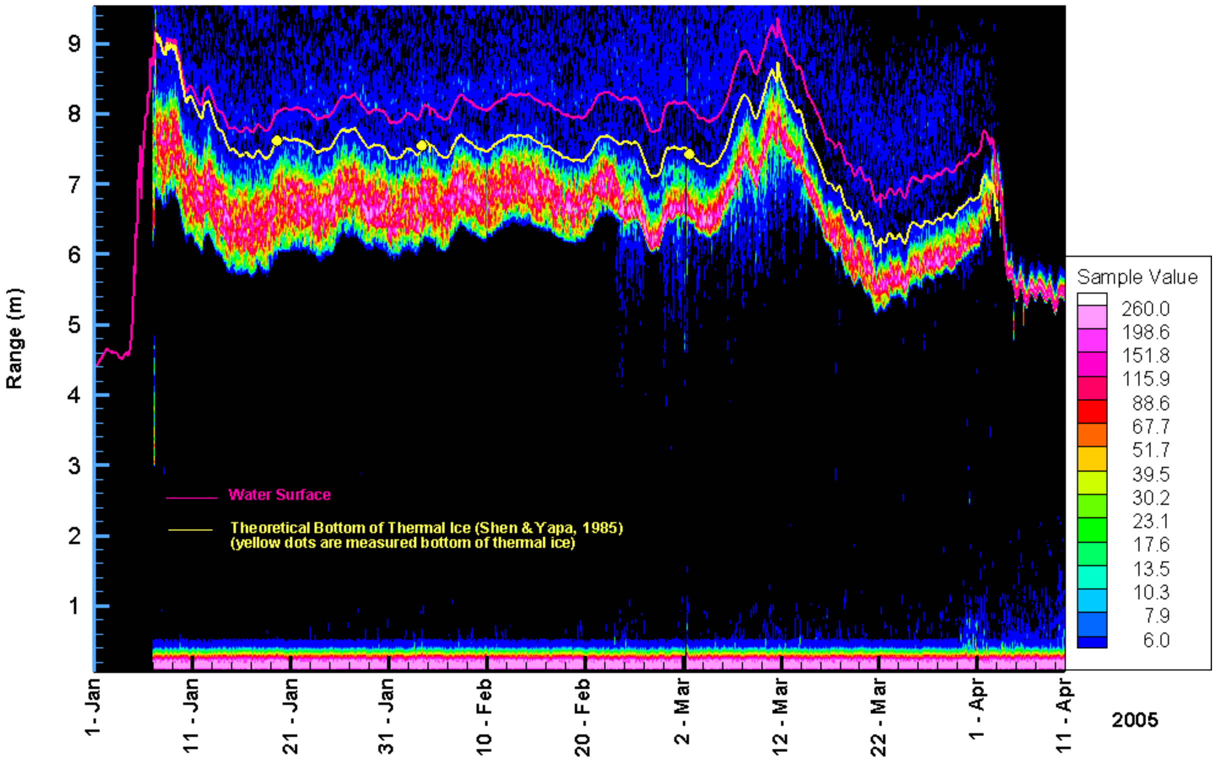
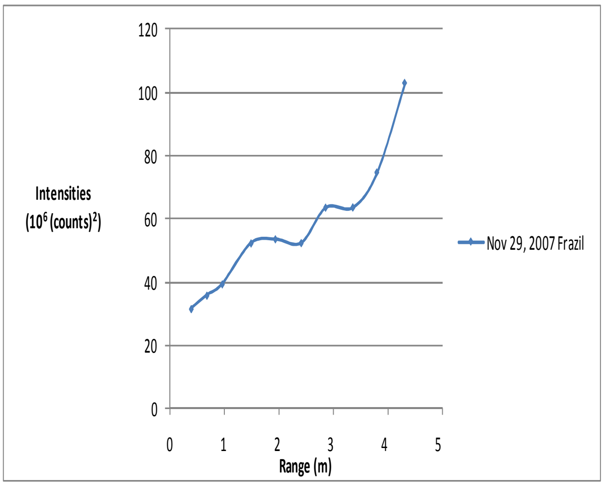


Figure 9: 235 kHz SWIP return strength as averaged over 2 hour intervals for the midwinter stationary ice covered period of January-April, 2005. Thermal break-up occurred on Apr. 3. Also shown are local water levels and positions of the modelled and measured (on 3 dates) bottoms of the thermal ice (Ref: Marko et al., 2006).



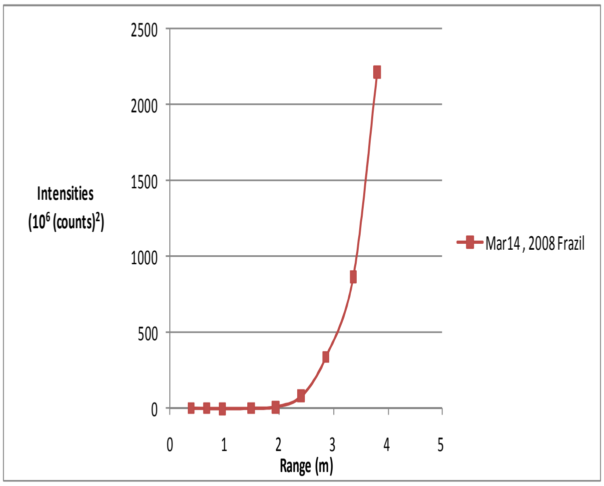


Figure 10: SWIPS relative intensities at 546 kHz as recorded in the absence of a stationary ice cover during a supercooling period (fig.. 10a on left) and under a stationary ice cover (fig. 10b on right).

SWIP intensities from frazil targets in a supercooled water column are typically smaller than those associated with frazil in the upper water column under a stationary, post-freeze-up, ice cover, as shown in Figure 10 (Marko and Jasek, 2010a). Note that the frazil ice present under the stationary ice cover is concentrated over a one metre range immediately below the more solid rive ice.

7. Conclusions and Future Directions

In this paper, the introduction of the Shallow Water Ice Profiler (SWIP) instrument for use in annual river ice monitoring programs on the Peace River has been described. The SWIPS has been developed for monitoring and study of river, lake and laboratory ice environments by providing detailed real-time quantitative information on the ice cover draft and the extent of frazil ice. The SWIP instrument provides two measurement modes: draft of floating ice in target mode and high resolution measurement of suspended ice crystals (SWIP intensities) over the full water column in profile mode.

During the past six winter seasons, these in-situ measurements on the Peace River with the SWIP have contributed significantly to further the understanding of the mechanisms and operational aspects of river ice processes (Jasek et al., 2005; Marko and Jasek, 2009). Real-time river ice draft measurements have been obtained season after season. The measured ice drafts included the thickness of a slush layer if present. The observation of frazil ice and eroded slush ice has been used in tactical decisions with respect to hydroelectric operations on the river.

From the multi-year Peace River measurement results obtained by BC Hydro, the detection of suspended frazil ice has been quantified in terms of mapping its temporal and vertical extent through the water column. These SWIP-based water column profile measurements of river ice, combined with its capabilities for draft measurements of the floating surface ice, provide the potential for reasonably complete quantitative tracking of all components of river ice systems.

The laboratory and field based research being conducted in collaboration with the University of Alberta is providing quantification and insights into river frazil ice processes (Ghobrial et al., 2009). SWIP instruments are also presently being used by research engineers and scientists in Japanese, Canadian and U.S. universities and research organizations.

The forthcoming introduction of the multiple-frequency SWIP instrument, with up to four discrete acoustic frequencies, promises to accelerate development of improved understandings of frazil crystal size and crystal concentration. This multi-frequency SWIP will provide new insights into the acoustic properties of slush ice and thermal ice.

It is anticipated that the SWIP instrument will lead to additional tactical inputs for hydroelectric plants and other operational applications on ice covered rivers.

Literature Cited

Chave, R.A.J., D.D. Lemon, D.B. Fissel, L. Dupuis and S. Dumont, 2004. “Real-Time Measurements of Ice Draft and Velocity in the St. Lawrence River Proceedings of Oceans 2004” Conference, Kobe, Japan. IEEE Press.

Fissel, D.B., J.R. Marko and H. Melling, 2008. Advances in upward looking sonar technology for studying the processes of change in Arctic Ocean ice climate. Journal of Operational Oceanography: 1(1), 9-18.

Ghobrial, T.R., M. Loewen and F. Hicks, 2009. “Frazil Ice Measurements Using the Shallow Water Ice Profiling Sonar”. Paper presented at the CRIPE Workshop on the Hydraulics of Ice Covered Rivers, St. John's, Newfoundland (Canada) June 14 - 17, 2009.

Jasek, M, J. R. Marko, D.B. Fissel, M. Clarke, J. Buermans and K. Paslawski, 2005. Instrument for detecting freeze-up, mid-winter and break-up ice processes in Rivers. Presented at 13th Workshop on the Hydraulics of Ice Covered Rivers. Sponsored by CGU HS Committee On River Ice Processes and the Environment, Hanover, NH, 34pp

Marko, J. and M. Jasek, 2008. Acoustic detection and study of frazil ice in a freezing river during the 2007-2008 winter. 19th IAHR International Symposium on Ice, Vancouver, Canada

Marko, J.R., D.B Fissel "Marine Ice Profiling: Future Directions" Presented at Icetech 2006 July 16-19, 2006, Banff, Canada

.

Marko, J.R., Fissel, D.B., Jasek M., 2006. Recent developments in ice and water column profiling technology. Proceedings of the 18th IAHR International Symposium on Ice, Sapporo, Japan, Aug 28 – Sep 1, 2006.

Marko, J.R. and M. Jasek, 2009. Estimation of frazil particle size and concentration from SWIPS measurements in the Peace River: an assessment of options and prospects. Proceeding of 5th CRIPE Workshop on River Ice St John's, Newfoundland, 13 p.

Marko, J.R. and M. Jasek, 2010a. Shallow water ice profiling measurements in a freezing river I: Methods and data characteristics. Submitted for publication in Cold Regions Science and Technology.

Marko, J.R. and M. Jasek, 2010b. Shallow water ice profiling measurements in a freezing river II: Observations and measurement results on suspended particulate (frazil) ice. Cold Regions Science and Technology.

Melling, H. and D.A. Reidel, 1995. The underside topography of sea ice over the continental shelf of the Beaufort Sea in the winter of 1990. J. Geophys. Res. **100(C7)**: 13641-13653.

Urick, R. J. 1983. Principles of Underwater Sound, 3 ed. McGraw-Hill, New York, USA.

Addendum

Converting counts to absolute backscatter and sound intensity -The Sonar Equation

The SWIP instrument applies a time varying amplification to the received voltage from the acoustic transducer to account for acoustic spreading and absorption in the water. Within the instrument, this amplified voltage is recorded using a 16-bit analog to digital converter as Counts (N) ranging from 1-65,535 (1-255 for the previous 8-bit model).

This section will describe how the data recorded by the SWIP may be related to the volume backscattering strength and sound intensity; it will not deal with echoes from discrete targets. The SWIP transmits an acoustic pulse of duration τ (pulse length) into the water and then records the intensity of sound arriving at its transducer as a function of time after the transmission. The echo is recorded at discrete intervals set by the digitization rate (64 kHz or 40 kHz); the intervals define cells, whose range away from the instrument is given by:

$$R = c \cdot t/2 \tag{S-1}$$

Where: R is in metres,
c is the speed of sound in the water (in m/s) and
t is the time (in seconds) after transmission at which the echo arrives.

Volume backscattering strength is usually denoted as S_v, defined as $10 \cdot \log_{10}(s_v)$, where s_v is the ratio of the intensity of the sound scattered from a unit volume, 1 metre away from the volume, to the intensity of the incident sound wave. Intensity of sound is defined as the power per unit area.

The intensity of the sound arriving at the receiver may be related to S_v by the sonar equation for volume reverberation (see, for example, Urick, 1983):

$$EL = SL - 40 \log R - 2 \cdot \alpha \cdot R + S_v + 10 \log V \tag{S-2}$$

Where: EL is the received intensity, measured in decibels, the logarithms are base 10,
SL is the transmitted signal intensity, measured in decibels,
α is the sound absorption coefficient (in decibels/metre) and
V is the volume insonified by the pulse at range R.

The spreading and absorption losses are represented by $40 \cdot \log R$ and $2\alpha \cdot R$ terms respectively.

V is defined as: $$V = ½ \cdot c \cdot \tau \cdot \Psi \cdot R^2 \tag{S-3}$$

Where: Ψ is the solid angle of an ideal conical beam equivalent to the integral of the actual beam pattern, and
τ is the duration of the acoustic pulse (seconds)

The transducer converts the sound pressure at the receiver to an electrical signal, which then passes through the instrument's signal conditioner and digitizer. The recorded counts, N, are related to the sound pressure at the receiver by:

$$N = P \cdot V_R \cdot g(2R/c) \tag{S-4}$$

Where: P is the received sound pressure level in micropascals (μPa)
V_R is the voltage response of the transducer to a reference pressure wave of 1 μPa and
g(2R/c) is the total amplification applied to the signal as a function of time.

The Voltage response is usually listed in the form of OCV which is the transducer receiving response in decibels ($OCV = 20 \cdot \log(V_R)$ or $V_R = 10^{OCV/20}$). The received sound pressure level is usually expressed as the received intensity measured in decibels ($EL = 20 \cdot \log(P)$ or $P = 10^{EL/20}$). Hence,

$$N = 10^{((EL+OCV)/20)} \cdot g(2R/c) \tag{S-6}$$

The total amplification includes the scaling factor for the analog to digital conversion and a time-varying component. It is useful to express the receiver gain expressed in decibels ($G(2r/c) = 20\log[g(2R/c)]$).

The sonar equation may then be solved for the volume backscatter at range R to give:

$$S_v = 20\cdot\log N - G(2\cdot R/c) - OCV - SL + 20\cdot\log R + 2\cdot\alpha\cdot R - 10\cdot\log(\tfrac{1}{2}\cdot c\cdot\tau\cdot\Psi) \qquad \text{(S-7)}$$

The time-varying component of the gain is designed to compensate approximately for the effect of the spreading and absorption losses in the sonar equation ($G(2\cdot R/c) \cong 20\cdot\log R + 2\cdot\alpha\cdot R$). Additionally, OCV, τ, Ψ, and SL are constants. Hence, the volume backscatter is more or less proportional to the logarithm of the recorded counts, N:

$$S_v \propto 20\cdot\log N \qquad \text{(S-8)}$$

By substituting for $S_v = 10\cdot\log_{10}(s_v)$, it is possible to obtain:

$$s_v \propto N^2 \qquad \text{(S-9)}$$

where: s_v is the ratio of the intensity of the sound (power per square unit) scattered from a unit volume to the intensity of the incident sound wave and N is signal amplitude measured in counts by the instrument.

If a more precise measure is desired, transducer calibrations for SL, OCV and Ψ, and electronics specifications for instrument gain ($G(2\cdot R/c)$) may be provided. Computations with such estimates are usually accurate to within ±2 dB. Greater accuracy values require direct calibration measurements of the specific instrument.