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Frazil Monitoring by Multi-frequency Shallow Water Ice Profiling Sonar (SWIPS): Present Status

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A methodology is outlined for extracting estimates of frazil particle sizes and concentrations as a function of height in the water column. It is based upon the use of SWIPS data acquired near-simultaneously at two or more acoustic frequencies at adjacent river bottom locations. The approach is tested on data acquired during three representative, but generically different, frazil appearance intervals. These intervals included: a pre-freezeup-supercooling event and "normal" and "anomalous" post-freezeup conditions. Distinctions among these alternatives are made in terms of the estimated particle sizes and concentrations and their dependences upon height in the water column or, in the post-freezeup case, on distance below the ice cover undersurface. The quality and content of the results are tentatively related to details of seasonal river processes but are, primarily, used to demonstrate current SWIPS capabilities and to identify improvements needed for fuller applications to outstanding frazil generation and transport issues.

1. **Introduction and Background.**

A prominent output of BC Hydro's Peace River SWIPS monitoring programs (Marko and Jasek, 2010a) has been the demonstration of nearly continuous, highly variable, acoustic backscattering from water column frazil during the freezeup period (i.e. under a locally stationary ice cover). Simple associations between the strength of this backscattering and the amount of frazil in the water column suggested that downstream ice transport beneath the ice cover was comparable to or larger than levels attained in the most intense but only episodic pre-freezeup supercooling periods. Characteristic differences in pre- and post-freezeup profiles of acoustic power returns from individual pulses or "pings" are readily apparent (Figure 1) in the squared SWIPS output amplitudes observed as functions of height in the water column above the upward-looking SWIPS transducer. Specifically, prior to freezeup, frazil intervals are characterized by significant return intensities from all levels in the water column, with intensities rising, very roughly, linearly with height, with possible mid-level flattening . After freezeup, returns tend to come from targets in the upper half of the water column, rising sharply near the ice cover. As well, post-freezeup intensities tend to show relatively strong correlations on time scales shorter than diurnal with local water levels and speeds and, to a lesser extent, with air temperatures or solar radiation levels. Typical correlations are illustrated relative to water levels in Figure 2.

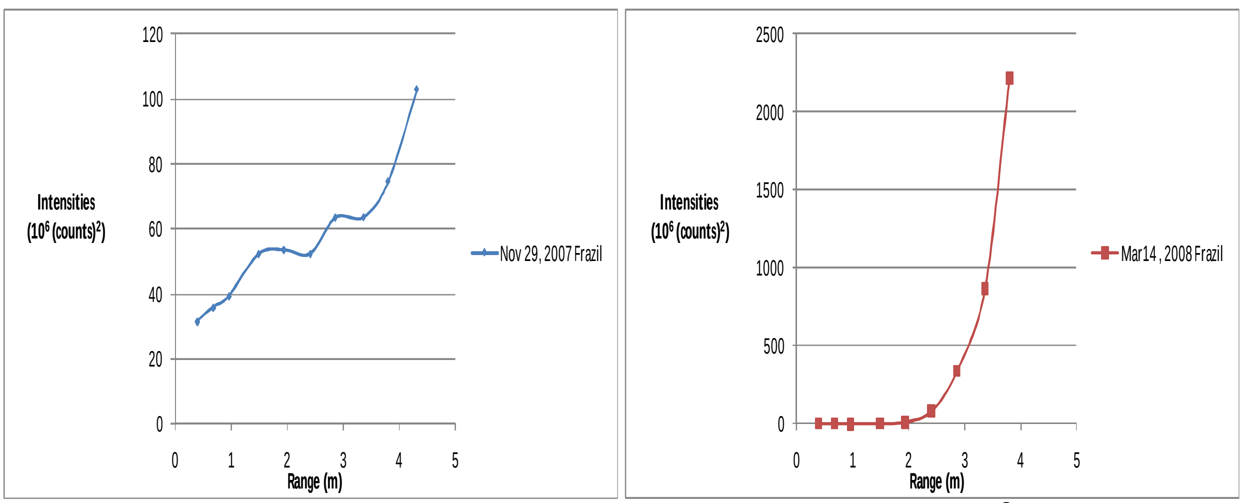

Figure 1. Average SWIPS Intensities, defined as (SWIPS return strength)^2, during typical pre-freezeup- (left) and post-freezeup- (right) intervals.

Quantitative understandings of the nature and origins of frazil variability both prior to and after freezeup are a major research and river management objective. In the former case, interests are focused on models of frazil and anchor ice growth and their effects on submerged structures and water flow intakes. Relevant issues after freezeup concern frazil transport at and below the ice cover and its role in ice jam formation, flooding and, more generally, in the evolution of the seasonal ice cover. In all cases, useful applications of SWIPS results require translation or conversion of acoustic backscattering data into directly river-relevant quantities such as frazil particle concentrations and sizes.

This step involves extracting measures of backscattering target strength per unit volume of the insonified water column in terms of the volume backscattering coefficient S_{vol} : a quantity which is proportional to the square of the SWIPS output signal. S_{vol} can be expressed in the form:

$$S_{vol} = \sum_{i=1}^{n} (N_i \sigma_i), \qquad (1)$$

where N_i and σ_i denote, respectively, the volumetric concentration (number of particles/volume)and backscattering cross sections (with dimensions of area) of individual particles in each of i = 1,....,n categories. The individual particle cross sections are measures of the fraction of acoustic power incident upon a particle which is backscattered toward the transducer. S_{vol} is calculated directly from the SWIPS count output using transducer and system parameters such as gains, conversion efficiencies, beam widths etc. Unfortunately, applications of Eq. 1 to any given measured value of S_{vol} can yield an infinite number of possible combinations of particle sizes and concentrations. We have previously (Marko and Jasek, 2009, 2010b) tried to reduce this ambiguity by imposing additional requirements which allowed treatment in terms of an inverted suspended sediment model. Unfortunately, this approach is restricted to the post-freezeup period and introduces the unlikely assumption that mean particle size is independent of height in the water column.

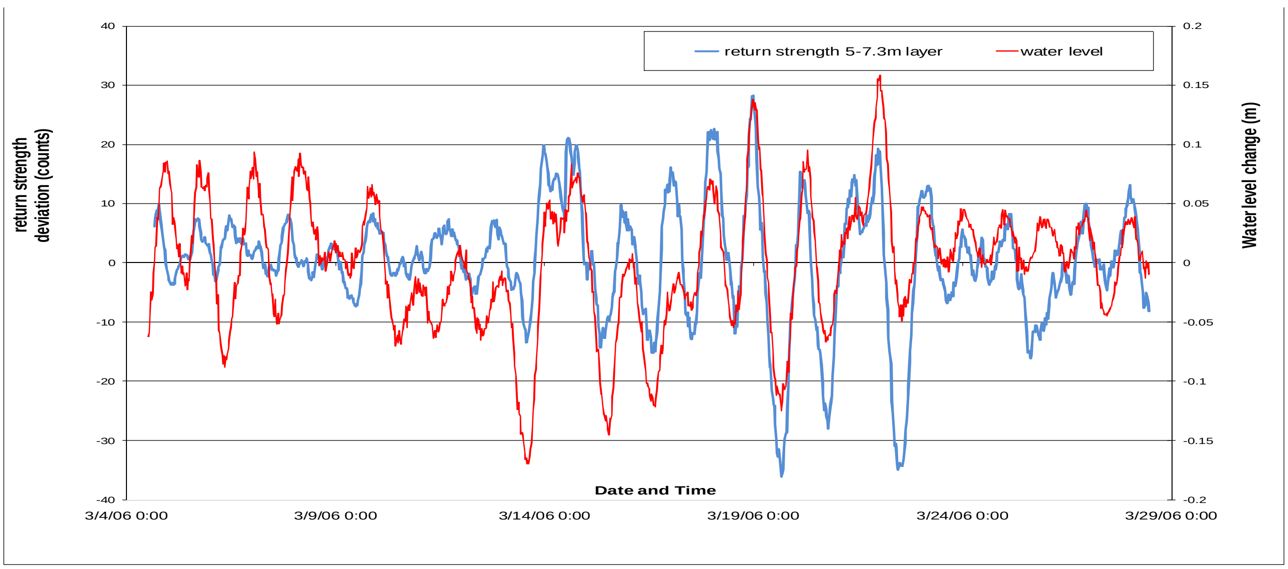

Figure 2. Average SWIPS returns at heights between 5 and 7.3 m and local water levels filtered to remove variability on time scales > 24 hours.

The present work offers a slightly more realistic methodology which, nevertheless, imposes its own simplifications which:

1. Treat the wide range of discoid and irregularly-shaped frazil particle sizes in terms of an equivalent population of spherical particles at a given range or height, h, in the water column in terms of single values of diameter, d(h) and concentration N(h); and

2. Assume the individual particle cross sections, σ_i , can be replaced at any given height by a value $\sigma(h)$ which is linked directly to the corresponding value of d(h) through the well established Rayleigh scattering relationship (Marko and Jasek, 2010a).

The first simplification lets us replace the sum in Eq. 1 for any height, h, by a simple product of one particle concentration and one cross. The second assumption draws on the Rayleigh relationship plotted in Figure 3 which dictates that, as long as $(2\pi/\lambda)(d(h)/2) < 1$ (where λ is the acoustic wavelength), particle cross sections are closely proportional to both the 4th power of acoustic frequency and the 6th power of particle diameter. The latter dependence means that backscattering signals tend to be dominated by the largest particles present in significant numbers. This dominance is very convenient for our purposes since, for similar reasons, these same particles are also the dominant contributors to the frazil-occupied fraction of the water column. Moreover, the availability of an only slightly weaker dependence of cross sections on frequency provides a mechanism for estimating both particle concentration and diameter at any height from corresponding measurements of $S_{vol}(h)$ at two different acoustic frequencies. In other words, the Rayleigh relationship allows us to represent the cross section at these frequencies in terms of a single diameter parameter. The value of this parameter, d(h), as well as a corresponding common concentration, N(h), can, thus, be uniquely determined from experimental estimates of $S_{vol}(h, \gamma)$) at any two (preferably well separated) measurement frequencies γ_1 and γ_2 (where $\gamma = c/\lambda$ and c= denotes the sound speed). In our applications, the relationship between cross section and particle diameter was derived from the solid-line curve in Figure 3 except for values ka = kd/2 > 1 where the indicated oscillations where replaced by the horizontal broken line relationship corresponding to $\sigma(d) = d^2/16$. The latter change reflects the reality of particle shape irregularities and the continuous distribution of particle dimensions which will average out the plotted oscillatory behaviour. Additional corrections also must be applied, which scale down the plotted cross sections by a multiplying factor of 0.05 (-13 dB) to account for the non-rigidity and density of ice particle targets (Clay and Medwin, 1977; Marko and Jasek, 2010b).

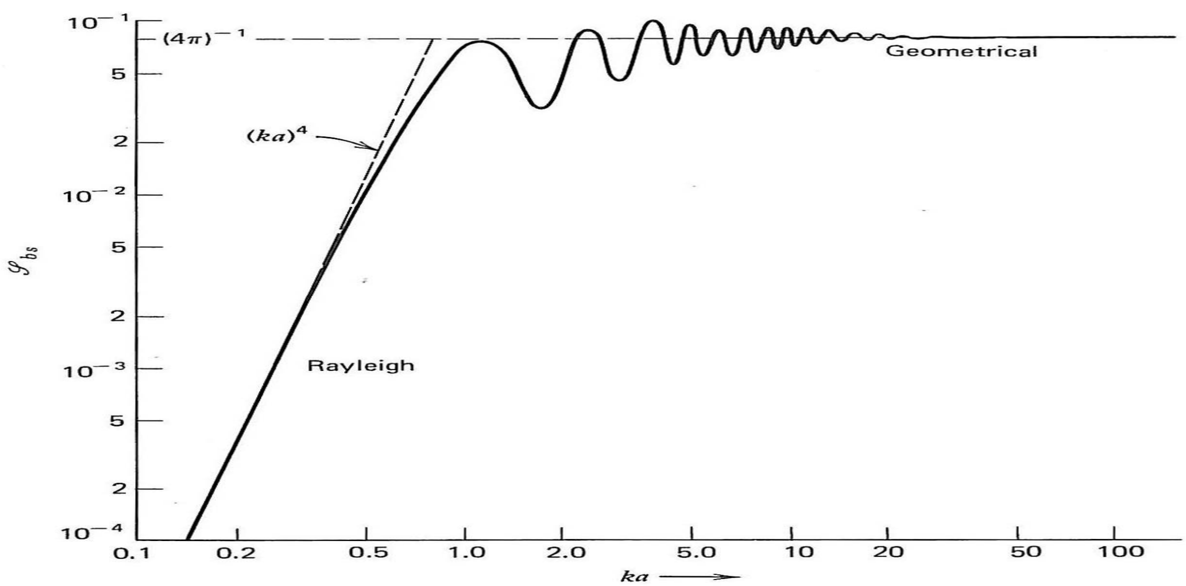

Figure 3. The backscattering function ($= \sigma_{bs}/\pi a^2$) vs. ka for a rigid sphere of radius a = d/2 and k= $2\pi/\lambda$ (Clay and Medwin, 1977).

Testing of this methodology has been an objective of the BC Hydro Peace River monitoring programs since the 2005-2006 field season when simultaneous SWIPS measurements were first attempted at more than one acoustic frequency. Unfortunately, loss of instrumentation and inadequate sensitivity at the lowest acoustic frequency impeded such testing until the appearance fortuitous conditions shortly after the 2009 freezeup and instrument modification in time for 2009-2010 deployment finally provided a suitable data base. Our extraction methodology is described in slightly greater detail in Section 2.1 below prior to presentation of preliminary test results in Sections 2.2 and 2.3 obtained on small portions of the data collected during, respectively, an early post-freezeup period in the 2008-2009 season and during pre- and post-freezeup frazil events in the 2009-2010 ice season. A closing Section 3 discusses these results and their implications for the future directions of the SWIPS measurement methodology and its applications.

2. Results

2.1 Extraction Method

The single term form of the summation in Eq.1 allows us to write the ratio of the volume backscattering coefficient as measure at heights, h, in the water column at frequencies γ_2 and γ_1, respectively as:

$$S_{vol}(\gamma_2,h)/S_{vol}(\gamma_1,h) = (N\,\sigma(\gamma_2,h))/\,(N\,\sigma(\gamma_1,h)) = \sigma(\gamma_2,h)/\sigma(\gamma_1,h). \quad (2)$$

Thus, under the right conditions (i.e. with frequencies γ_1 and γ_2 and with particle diameters, d, satisfying the $(\pi d/\lambda) < 1$), the ratios of $S_{vol}(\gamma)$ measured at a common height at two different frequencies give measures of the ratio of the corresponding cross sections at that height as calculated by Rayleigh scattering theory. Such cross sections can be derived for any given frequency and particle diameter from Figure 3 (after correction (Marko and Jasek, 2010a) to accomodate ice density and compressibility parameters). Their ratios are solely functions of particle diameter and the two measurement frequencies and, thus, define a unique value for particle diameter at any given combination of measurement height and frequencies. The resulting relationship for the pairing of the 235 and 546 kHz frequencies used in the Peace River studies, plotted in Figure 4, allows straightforward conversion of ratios $r = S_{vol}(\gamma_2,h)/S_{vol}(\gamma_1,h)$ into $d(h)$ values. The resulting diameter estimates can then be used to obtain a corresponding value of $\sigma(\gamma,h)$ at either measurement frequency to calculate $N(h)$ from the single term form of Eq. 1 and the measured value of $S_{vol}(\gamma,h)$ at that frequency. In this way, SWIPS backscattering results provide estimates of both particle size and concentration at any level in the water column.

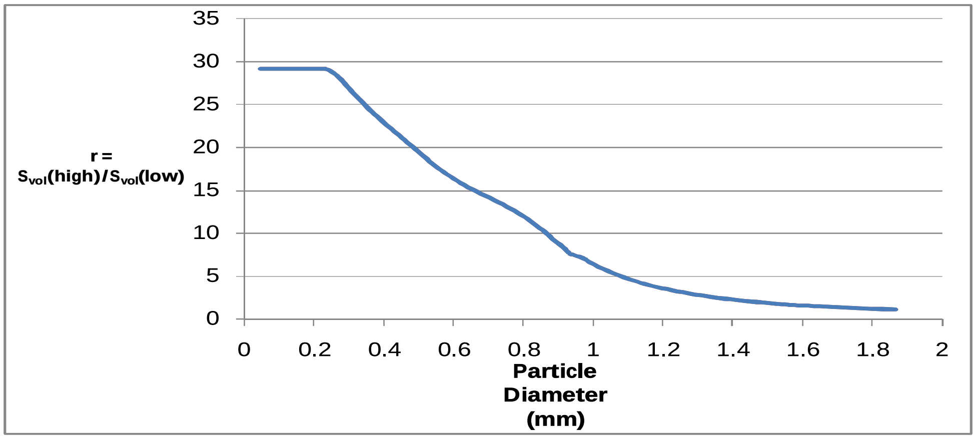

Figure 4. The dependence of ratio of kHz (high) and 235 kHz (low) acoustic frequency volume backscattering coefficients on spherical particle diameter in Rayleigh scattering theory.

2.2 Initial Applications of Extraction Methodology

As indicated above, prior to the 2009-2010 field season, the sensitivities of the 235 kHz SWIPS instruments deployed in the Peace River were insufficient to support testing of the proposed extraction approach. In the 2008-2009 deployment, however, exceptionally large low frequency return strengths were encountered during the first three weeks of the ice-covered season. The origins of this anomaly will be briefly considered below and, in more detail, by Jasek et al. (2010) but were immediately suspected to be indicative of larger than normal frazil particle sizes. The strengths of the returns at both frequencies steadily weakened during this "anomalous" period and, initially at least, showed little evidence of correlations with environmental parameters similar to those depicted in Figure 2. By late January, meaningful returns were only detectable at the higher, 546 kHz, SWIPS acoustic frequency and the correlations with environmental parameters were again very apparent. Data from the anomalous period provided dramatic evidence of previously suspected (Marko and Jasek, 2010b) complications introduced by return strength dependences on both particle size <u>and</u> concentration. The situation is illustrated in Figure 5 by S_{vol} data as measured at both 235 kHz and 546 kHz on Jan. 10 and Jan. 12. It can be seen that, while volume backscattering at the lower frequency decreased over the intervening interval, S_{vol} at 546 kHz increased over this same period. Clearly, measurements at either frequency were insufficient in itself for characterizing corresponding changes in particles size and number.

Compatible, combined, use of data gathered at both frequencies was complicated by the physical separation (by several m) of the deployed instruments under a deformed ice cover of considerable thickness. Given the anticipated (Marko and Jasek, 2010b) close linkage between the detected water column frazil and the ice cover undersurface, it was convenient to offset the range readings of the high frequency unit to achieve identical low and high frequency measures of elevations of this surface relative to the respective instrument transducers. This adjustment facilitated calculations of high frequency/low frequency S_{vol} ratios as a function of common distances below the local ice cover undersurfaces.

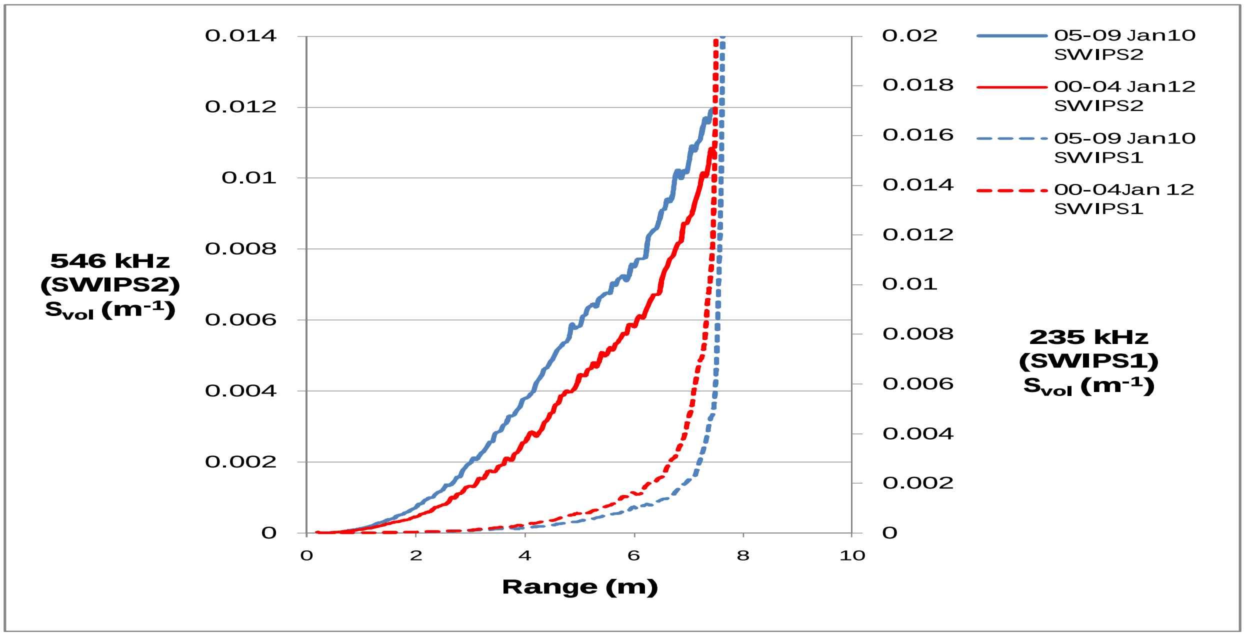

Figure 5. Time averaged values of S_{vol} as measured with the low frequency (SWIPS1) and high frequency (SWIPS2) instruments as a function of vertical range relative to the SWIPS1 transducer face for the 05:00-09:00 January 10, 2009 and 00:00-04:00 January 12, 2009 time intervals.

The relevant backscattering results are presented in Figure 6 in terms of S_{vol} values at each frequency as well as their ratio as functions of the adjusted common range parameter for the two periods depicted in Figure 5. The plots also include the estimated contemporary ranges to the ice/water interface. The low frequency returns are seen to have been weak relative to their high frequency counterparts except near the ice undersurface where their longer wavelengths allowed returns to more closely approximate reflections from a smooth surface and, hence, yielded larger returns. The high frequency data also deviated strongly from the sharply upturning form depicted in Figure 2 as representative of post-freezeup returns. Instead, the range dependence is roughly linear, resembling in form that depicted (Figure 1) for pre-freezeup periods. The plotted ratios, S_{vol}(546 kHz,h)/S_{vol}(235 kHz,h) provided the basis for estimating particle diameters from the relationship depicted in Figure 4. Expectations were that this ratio should rise from small values at maximum range in the water column (i.e. at the ice undersurface) to much larger values deeper in the water column to reflect corresponding expected decreases in particle size. For the utilized frequencies, this ratio should not have exceeded 29.14. This last expectation was violated late in the anomalous period when the low acoustic frequency returns from the lower water column became extremely weak. This result was traced to deficiencies in the “envelope detecting” portion of the SWIPS’ acoustic board which underestimated weaker signal levels. In any case, problems of this type were restricted to the last half of the anomalous January, 2009 period and to depths more than 2 m below the ice/water interface. Subsequent increases in the gain of the low frequency unit gain prior to the 2009-2010 measurements (Section 2.3) appears to have eliminated this problem.

Particle diameter, volumetric concentration and fractional volume estimates drawn from the well-behaved portions of results in Figure 6 clarified the origins of the opposing low and high frequency S_{vol} trends noted in Figure 5. These data (Figure 7a-c) show that changes in S_{vol} over the Jan. 10-12 time interval arose from significant increases in particle size coincident with decreases in particle concentrations. Overall, the fractional volume (the fraction of water column volume at any level occupied by ice) changed little over the intervening interval. These results confirmed initial expectations that the anomalous detectability of low acoustic frequency SWIPS returns was a consequence of the large sizes of the particles: with particles having diameters in excess of 0.8 mm having been found down to depths 3 to 4m below the ice cover during the measurement periods. It is significant that the estimated diameters in the upper reaches of the water column exceed the 0.8 mm limit associated with the start of deviations from the strong Rayleigh scattering size and diameter dependences at 546 kHz. This last result suggests that more accurate frazil SWIPS characterizations in similar periods would be achieved using a lower "high" acoustic frequency.

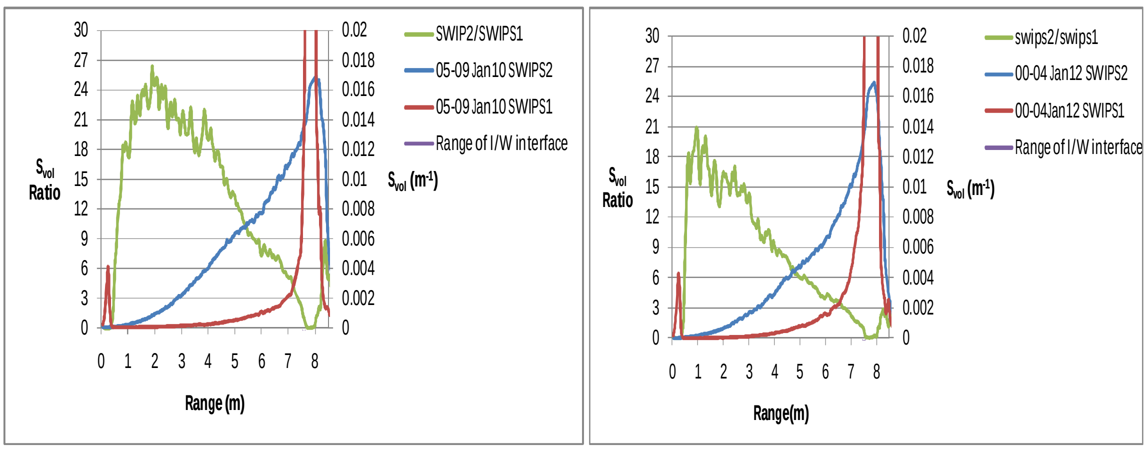

Figure 6. Plots of time averaged S_{vol} as measured for 05:00-09:00 Jan. 10, 2009 (left) and 00:00-04:00 Jan. 12, 2009 (right) time intervals at both low (SWIPS1) and high (SWIPS2) acoustic frequencies as functions of vertical range above the face of the SWIPS1 transducer. Plots include the ratios of the two S_{vol} quantities and the estimated range of the ice cover undersurface.

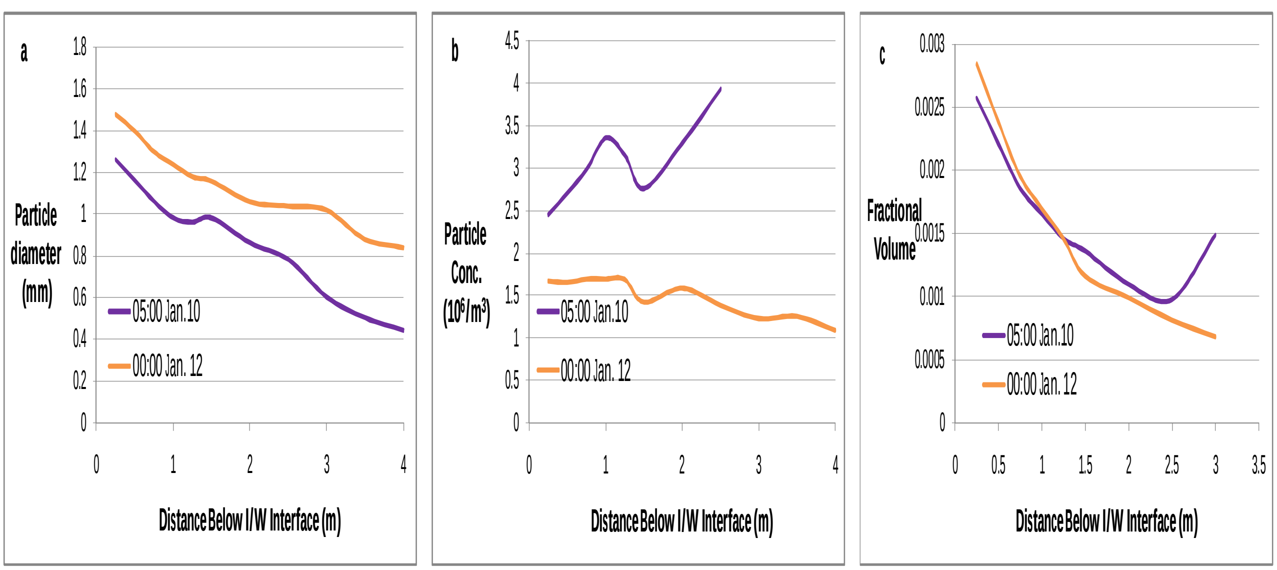

Figure 7. Estimates of: a) diameters; b) volumetric concentration; and c) fractional volume associated with idealized spherical frazil particles as a function of distance below the ice/water interface for the indicated Jan. 10 and 12 time periods.

2.3 Applications to 2009-2010 Data

Very small but representative portions of the 2009-2010 seasonal data record were analyzed to provide the first substantial tests of our extraction methodology on data gathered both prior to the freezeup period and during a “typical” post-freezeup interval. Our definition of “typical” in this contex requires both maximum particle sizes being somewhat smaller than those identified in the foregoing “anomalous” interval and the presence of previously observed correlations between SWIPS return strengths and environmental parameters. Again, this additional testing was enabled only by enhancement of the sensitivity of the low acoustic frequency SWIPS instrument.

Pre-freezeup test data were drawn from an hour-long Dec. 20, 2009 time interval associated with the most intense portion of a, roughly, 24 hour period of consistently intense SWIPS returns. Data from both instruments yielded near-identical values for the range to the water/air interface: simplifying combined use of low and high frequency profile data and allowing presentation of extracted size and concentration results as a function of a common range or height variable. The averaged high frequency S_{vol} profile for this interval (Figure 8) exhibited the rough linearity and mid-water column flattening typical of pre-freezeup data (Figure 1). More dramatically, at both low and high frequencies, S_{vol} values were one to two orders of magnitude below those associated with the anomalous post-freezeup 2009 period. This intrinsic weakness of pre-freezup frazil returns accounts for past inabilities to acquire usable amounts of corresponding low acoustic frequency data. The higher sensitivity of the 2009-2010 low frequency unit enabled extractions of the particle size, volumetric concentration, and fractional volume estimates plotted in Figure 9a,b. These estimates were derived from SWIPS returns gathered at 5 cm range or height intervals prior to smoothing with a 5 point running average filter. The latter step facilitated comparisons with the 25 cm interval data presented in Section 2.2. The noisier character of the pre-freezup data curves reflect, in part, the shorter averaging period (1 hr vs. 4 hrs) used in the 2009-2010 analyses.

The results in Figure 9 show the prevalence of smaller particle sizes, with diameters approaching the, roughly, 0.8 mm value marking the breakdown of the Rayleigh parameter dependences (at 546 kHz) only in the upper 1 m of the water column. At greater depths, the falloff in size is compatible with a previous estimate based upon 2006 Peace River data (Marko and Jasek, 2010b) which anticipated the presence of 0.4 to 0.5 mm particle diameters 2 m above the transducer. The concentrations of particles in the upper two meters, associated with diameters > 0.6 mm are approximately $10^5/m^3$. The substantial 546 kHz returns from greater water column depths (Figure 8) and their inferred association with still smaller particles implies the presence of an accompanying increase in volumetric concentrations (Figure 9a). The resulting peak in the estimated concentration curve (not fully displayed to allow representation of the upper water column results) corresponded to about $4 \times 10^7/m^3$. The fractional volume results (Figure 9b) indicate that the highest fractions of water column frazil content occurred at heights of 1 to 2m above the transducer. These results are not in accord with simulations (Hammar and Shen, 1995) which predicted fractional volumes increasing monotonically and sharply with height in the water column. This disagreement is a possible consequence of model-neglected complexities such particle subdivision (Marko and Jasek, 2010b).

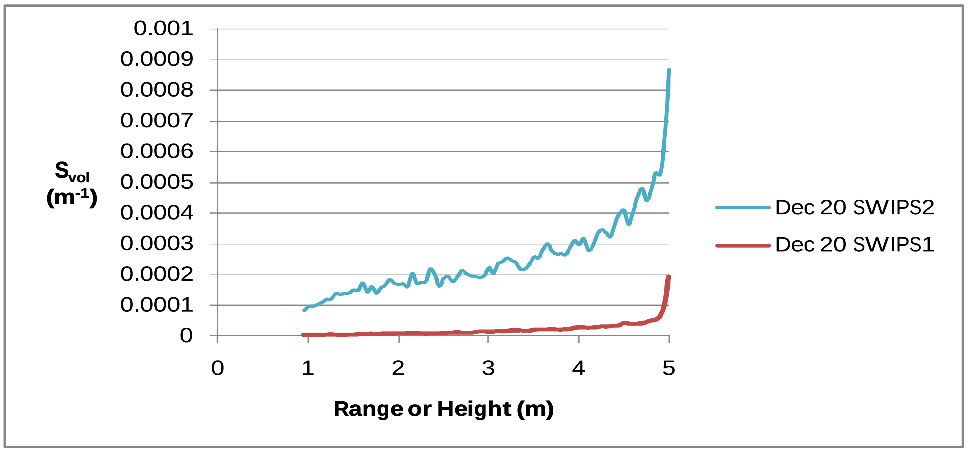

Figure 8. Average S_{vol} estimates for a one hour Dec. 20, 2009 time interval.

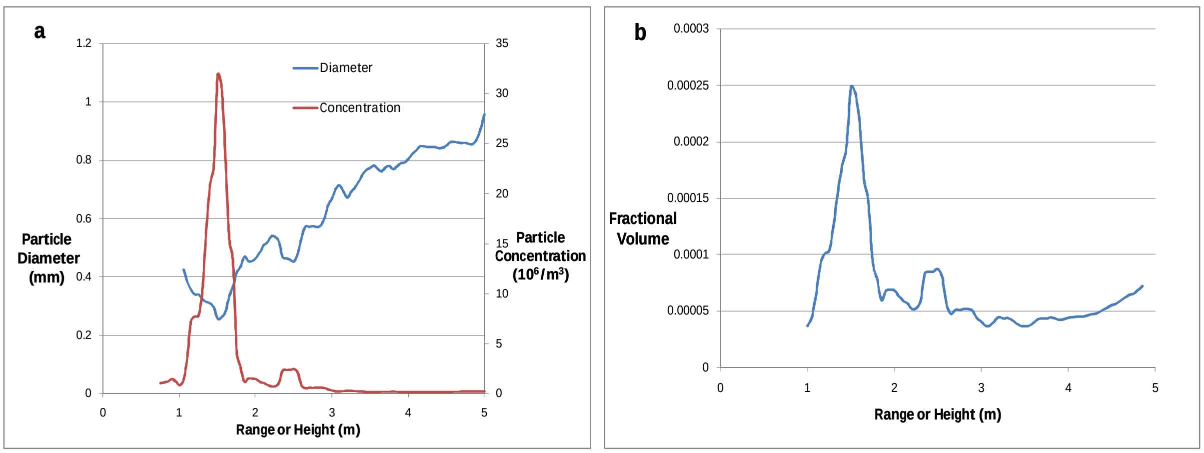

Figure 9. Plots of: a) average particle diameter and volumetric concentration; and b) fractional volume associated with frazil during the one hour Dec. 20, 2009 pre-freezup measurement period.

Equivalent post-freezeup results were also generated from data collected between 00:00 and 01:00, January 28, 2010. The volume backscattering coefficients for this period, roughly an order of magnitude larger than those associated with the above pre-freezeup results, were significantly smaller than those attained in the anomalous 2009 post-freezeup period. Again, the latter difference accounts for the absence of detected low acoustic frequency returns in the "non-anomalous" portion of the 2009 post-freezeup season. As in Section 2.2, the expected linkages between frazil properties and the overlying ice cover favoured presentations of S_{vol} data which facilitate joint extraction of frazil parameter values as a function of distance below the ice/water interface. Consequently, the ranges/heights in the high frequency results in Figure 10 were elevated from their measured values by 0.85m (the difference between the ranges to the ice undersurface at the high and deeper (relative to the latter interface) low frequency measurement sites). An additional complication in this case was the history of the ice cover above the measurement sites which included a period of localized open water which had refrozen and was still thickening at the time of our measurements. The stronger high frequency returns from the interface itself relative to the lower acoustic frequency ran counter to the anomalous 2009 post-freezeup period results and probably reflected differences in ice topography and texture. Nevertheless, the particle size, concentration and fractional volume data in Figure 11a,b appear robust: with particle diameters falling smoothly with depth from values

> 1 mm to about 0.2 mm over the upper 3 m of the water column. Corresponding particle concentrations fell similarly from slightly above $10^6/m^3$ to approximately and just below $10^5/m^3$. The much larger concentration peak of smaller particles depicted at depths of 3 to 4 m could be an artefact of the separation of the two measurement sites but, in any case, makes a negligible contribution to the water column's fractional ice content (Figure 11b). In this case, the high frequency S_{vol} curve (Figure 10) is a crudely representative descriptor of the fractional ice volume parameter. The data for the latter parameter suggest that only at the very top of the water column did the frazil content in this interval begin to approach the levels reached in the upper 2 to 3 m of the water column in the anomalous 2009 time intervals. Nevertheless, when integrated over the full height of the water column, the water column ice content in this period appears to have been slightly larger than that similarly deduced above for the intense pre-freezeup period.

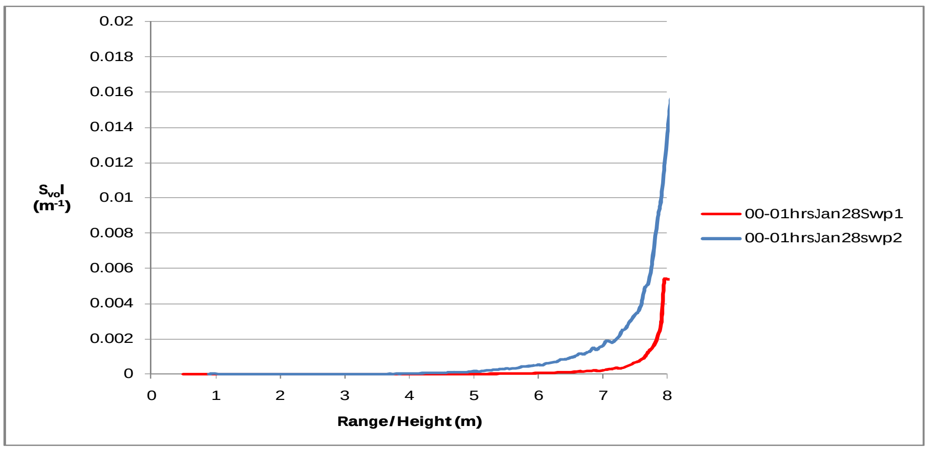

Figure 10. Average S_{vol} estimates for a one hour 00:00-01:00 Jan. 28, 2010 time interval.

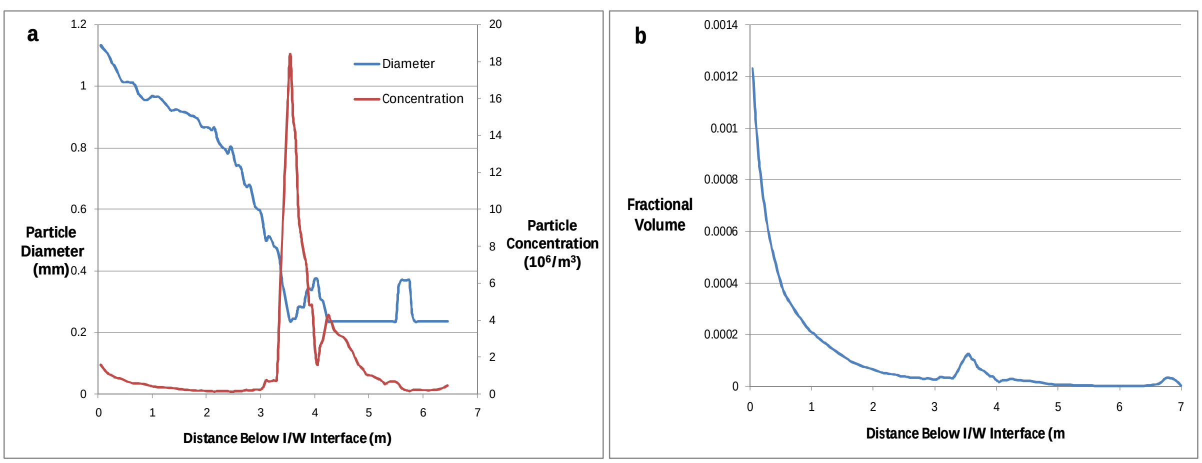

Figure 11. Plots of: a) average particle diameter and volumetric concentration; and b) fractional volume associated with frazil during a one hour Jan. 28, 2010 post-freezeup measurement period.

3. Discussion and Conclusions

Section 2 describes applications of a simple characterization methodology to data collected during three recognizably distinct types of frazil occurrence intervals. Two of the intervals, treated in Section 2.3, are common features of a freezing river, corresponding, respectively, to active particle growth in a supercooled water column and to passive (non-accreting) particles present after freezeup beneath a stationary ice cover. In the latter case, acoustic backscattering from ubiquitous frazil particles has been previously shown to be statistically linked to changes in river flow and thermal factors. The third kind of interval, treated in Section 2.1, spanned only a small fraction of the post-freezeup period but was associated with the largest rates of downstream frazil ice transport. There is some evidence that the latter period was typical of early portions of freezeup periods associated with weak or absent correlations with river flow and thermal parameters (Marko and Jasek, 2010b). The timing of such “anomalous” events and subsequent ice cover data gathering suggests possible connections to active frazil growth in areas upstream of an advancing ice front and could be important in determining the character and slush ice content of the seasonal ice cover.

Our analyses of small amounts of representative data appear to provide a basis for making basic distinctions among these frazil forms. Thus, frazil particles in supercooled water appear to have smaller physical dimensions than their post-freezeup counterparts and are more widely distributed throughout the water column. Supercooled period size estimates are consistent with the peak frazil contents, as expressed in terms of fractional occupation of water column volume, which arise from exceptionally high concentrations of smaller than average particles in the lower half of the water column. Post-freezeup frazil populations in both the anomalous and non-anomalous categories tend to have particle diameters close to or greater than 0.8 mm in the upper half of the water column, with particle size and concentration both rising toward the ice cover undersurface. These rises are sharpest in the non-anomalous case where most of the water column ice content is confined within 1m of the ice/water interface. Nevertheless, at least in the example considered here, the total integrated water column ice content in this case tends to be comparable or, probably, larger than the levels associated with the most intense portions of any given pre-freezeup frazil event. The anomalous post-freezeup period was found to be associated with the highest levels of water column ice content and transport due to the presence of high ($>10^6$ /m^3) concentrations of particles with diameters close to and greater than 1 mm down to depths of 3 to 4 m.

Refinements and extensions of this description and progress toward understanding the factors controlling frazil variability will require comprehensive extractions of frazil population data on the, still unknown, time scales associated with changes in particle size and number. Specifically, correlations of size and concentration parameters with environmental factors are likely to offer the clearest clues as to underlying variability mechanisms. Multifrequency SWIPS measurements are likely to be an essential tool in supporting such efforts and can be carried out concurrently with essential, and still largely absent, independent verification and calibration activities. Such activities have been initiated at the University of Alberta and, hopefully, will be incorporated into other SWIPS measurement programs. Particular needs are for independent comparisons with SWIPS-estimated frazil parameters and for further establishing the physical properties of frazil particles appropriate to Rayleigh theory descriptions. It is also of interest to quantify the errors introduced by simplifying frazil population descriptions into spherical particles with diameters and concentrations which vary with height in the water column.

Other steps toward improved measurements arise naturally out of the obvious shortcomings of initial multifrequency applications. Specifically, complications were encountered from both the physical separations of the low and high acoustic frequency instruments and from the fact that some of detected particles appeared to be large enough to have cross sections with weaker (non-Rayleigh)

(Figure 3) size dependences. The first of these problems introduced uncertainties into the critical computations of S_{vol} ratios, particularly under an ice cover of spatially varying thickness. In the second case, particle size estimates were, of necessity, based upon ratio values corresponding to the relatively flat right hand portion of the curve in Figure 4. This circumstance made diameter estimates very sensitive to small errors in the measured S_{vol} ratios. As well, the weaker cross section dependences undermine the assumption that a frazil population can usefully be described in terms of its larger members and, more generally, diminish the utility of acoustic frequency as a size-discriminator. These problems are currently being addressed in the design of an integrated multifrequency SWIPS unit which will include 4 co-located transducers operating at frequencies selected to optimize ratio relationships to the anticipated particle size regimes. Expansion of capabilities to more than two frequencies will also allow extraction of additional information defining the width and asymmetry of frazil particle size distributions.

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